IndoMS. J.M.E
Vol. 3 No. 2 July 2012, pp. 175-186

Rectangular Array Model Supporting Students' Spatial Structuring in Learning Multiplication

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Abstract

We examine how rectangular array model can support students' spatial structuring in learning multiplication. To begin, we define what we mean by spatial structuring as the mental operation of constructing an organization or form for an object or set of objects. For that reason, the eggs problem was chosen as the starting point in which the students could recognize such an arrangement. Geoboard was also be used as a tool to visualize the array. This research focused on a design research that was conducted in Surya Institute Program (SIP) in which investigated 12 Papuan students (between 10 and 11 years old) in connecting rectangular array model with the idea of multiplication. The result showed that rectangular array model indeed support the students to count things more efficient, able to see the structural similarities of arrays and created spatial structures for sets of objects.

Keywords: array, spatial structuring, multiplication.

Abstrak

Kami meneliti bagaimana *rectangular array model* dapat mendukung *students' spatial structuring* dalam belajar perkalian. Untuk memulai, kita mendefinisikan apa yang kita maksud dengan *spatial structuring* sebagai mental beroperasi dalam membangun sebuah organisasi atau bentuk untuk suatu obyek atau sekumpulan objek. Untuk alasan tersebut, permasalahan tentang telur terpilih sebagai titik awal pembelajaran dimana siswa bisa mengenali pengaturan tersebut. Geoboard juga digunakan sebagai alat untuk memvisualisasikan *the array*. Penelitian ini difokuskan pada desain penelitian yang dilakukan di Surya Institute Program (SIP) dengan menyelidiki 12 mahasiswa Papua (usia 10 dan 11 tahun) dalam menghubungkan *rectangular array model* dengan konsep perkalian. Hasil penelitian menunjukkan bahwa *rectangular array model* memang mendukung siswa untuk menghitung sesuatu menjadi lebih efisien, mampu melihat kesamaan struktural array dan menciptakan struktur spasial untuk suatu kesatuan benda.

Kata Kunci: *array*, *spatial structuring*, perkalian.

Introduction

Multiplication, as a concept and skills, is one of the learning objectives from grade 2 elementary school. It comes after basic addition and subtraction have been taught. This should make it possible for the students to conceptualize the idea of multiplying objects or figures. However, learning multiplication can be difficult for students who have difficulties in conceiving the meaning of multiplication itself. In addition, traditionally, the students are also asked to memorize the multiplication facts without knowing what multiplication is. Smith & Smith [6] indicated that they might have algorithmic skill but no mathematical knowledge of multiplication.

In Indonesia, learning multiplication often focuses in counting the number of equal groups of objects which leads the students to the idea of multiplication as repeated addition. Whereas, Mulligan [3] stated that there are four models which primarily apply to multiplication problems, namely equivalent sets (e.g. 3 tables, each with 5 children), multiplicative comparison (e.g. 4 times as many boys as girls), cartesian product (e.g. the number of possible boy-girls pairs), rectangular array (e.g. 4 rows of children).

Considering the need of developing students' spatial structuring and realizing that lack of research in Indonesia, we tried to design instructional activities which involved rectangular array as a model. Although array models are used to show multiplicative relationships, Outhread & Mitchelmore [4] revealed that the students may not see structural similarities of discrete arrays and arrays as a grid of contiguous squares, thus they may not connect an array of squares with multiplication. To link the array model to multiplication, we tried to guide the students to perceive that the rows are equal and correspond to equivalent groups. By using geoboard as the visual aid, the students were expected to connect array situation with multiplication. Besides, this research was also used a method of learning and teaching mathematics that is called "Matematika GASING" (*GAmpang aSyIk menyenaNGkan*) or easy, exciting and enjoyable mathematics which developed by the Surya Institute. This method helps students to learn math easily, whatever the person's background. Thus, the research question was formulated as follows.

"How can rectangular array model support students' spatial structuring in learning multiplication?"

Theoretical Framework

Basic Understanding of Multiplication

Building an understanding of the concept of multiplication requires developing a language for thinking about and describing multiplicative situations and their quantities. Particularly, students' experiences with multiplication occur in situation where there are a number of groups of objects having equal quantities. For example, counting the number of donuts in five dozens, they can multiply 5×12 because they know that each dozen has 12 donuts.

Smith & Smith [6] reported that drawing on the mathematics education literature for a basic understanding of multiplication leads to consider four interconnected concepts: (a) quantity, (b) multiplicative problem situations, (c) equal groups, and (d) units relevant to multiplication. Most of these understanding can be developed from the experiences using counting and grouping strategies to solve multiplication problems.

The Models of Multiplication Problems

As described by Mulligan [3], there are four models which primarily apply to multiplication problems, namely:

1. equivalent sets (e.g. 3 tables, each with 5 children),
2. multiplicative comparison (e.g. 4 times as many boys as girls),
3. cartesian product (e.g. the number of possible boy-girls pairs),
4. rectangular array (e.g. 4 rows of children)

Van de Wall, et al [8] mentioned that there are three models of operations, namely equal sets, array, and number line. The models for equal-group multiplication are shown in the figure below.

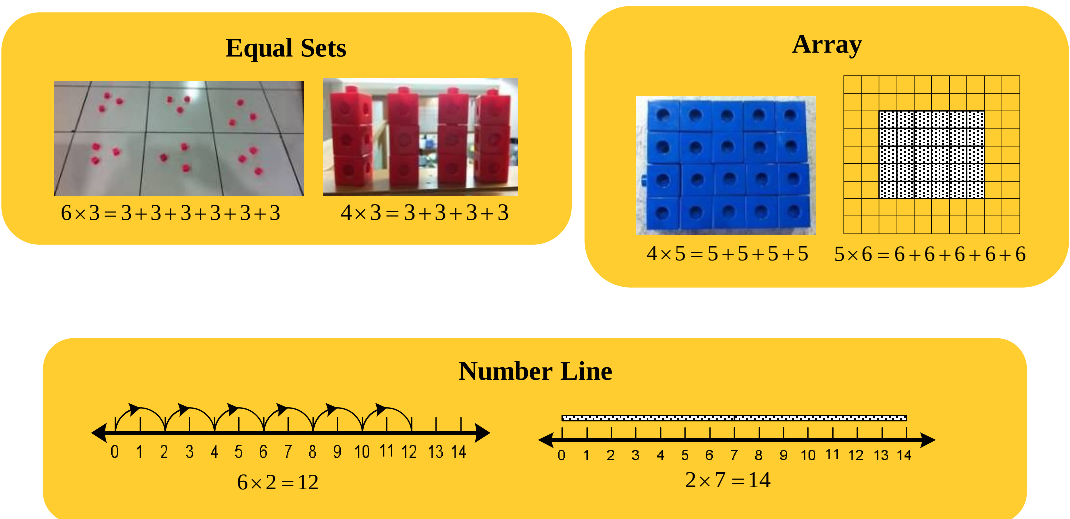


Figure 1. Models for equal-group multiplication.

At the beginning, students might use the same model such as sets and number lines for all four arithmetic operations. However, a model which is not generally used for addition but also important for learning multiplication is the array. An array in [8] is any arrangement of things in rows and columns, such as rectangle of square tiles or blocks. Array provides a representation of multiplication and may help the students who are visual learners. Students can construct rectangle consisting of four rows of seven pegs (figure 2) to model the multiplication fact 4×7. Array situation should also focus on the number of rows to model multiplication as repeated addition.

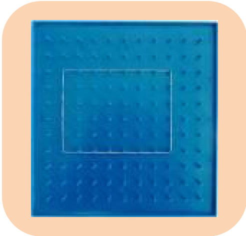


Figure 2. Geoboard as an aid to visualize rectangular array.

MATH GASING (GAmpang aSyIk menyenaNGkan) or E_3 (Easy, Exciting and Fun)

Based on personal communication with Prof. Yohanes Surya [7], the founder of MATH GASING, GASING method is a step by step method to achieve the goal of the learning. The emphasis of step by step method is the key word of GASING. GASING stands for Easy, Exciting, and Fun. Easy means the students are introduced to mathematical logic that is easy to learn and to remember. Exciting means the students have motivation which comes from themselves to learn mathematics (intrinsic factor). Fun is more in the direction of outside influences such as visual aids and games (extrinsic factor).

For instance, in learning addition, this method has the steps of learning so that the students are able to do the mental calculation. In this learning step, there is a critical point that must be passed. After passing the critical point, the way to master a mathematical topic will be more smoothly. Some critical points in learning mathematics are as follows.

- In learning addition, the students should master the sum of numbers less than 20.
- In learning multiplication, the students should master the multiplication up to 10×10.
- In learning subtraction, the students should master subtraction of 1 digit numbers.

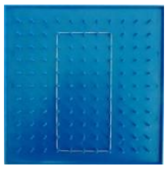
- In learning division, the students should master non-divisible numbers.

Research Method

The type of research that we used was design research [5]. Design research consists of three phases, namely developing a preliminary design, conducting teaching experiments, and carrying out a retrospective analysis. Twelve Papuan students in Surya Institute Program (SIP) were involved in this research. The students were about 10 to 11 years old. The data collected in this research were interviews with the students, classroom observations including field notes and students' works. After we collected all data, we analyzed these data in the retrospective analysis phase. Finally, we made conclusions based on the retrospective analysis. The instructional activities of learning multiplication using rectangular array model are as follows.

Table 1. Instructional activities of learning multiplication using rectangular array model.

Learning Goals	Mathematical Ideas	Activities	Students' Struggles	Students' Strategies
a. Students count the number of objects from the figure.	- Unitizing, e.g. five rows of six.	Counting the number of eggs. 	- To keep track the counting process when they use strategy of counting by ones.	- Counting by ones. - Skip counting forwards. - Repeated adding.
b. Students construct a rectangle using geoboard and count the number of pegs in the	Spatial structuring of array comes as a result of organizing actions on the sets of objects (pegs).	Counting the number of pegs which are formed from a rectangular array on the geoboard.	- To keep track the counting process when they use strategy of counting by ones.	- Counting by ones. - Skip counting. - Counting the number of pegs in each side then

geoboard.			- To understand how one peg can simultaneously be part of a row and a column.	multiply it (the idea of rows times columns)
c. Students draw rectangular array	Mental representation of array structure by abstraction from physical or pictorial models.	Drawing array structures which represent multiplication problems.	- Limited conceptions of array structure. - Difficult in differentiating rows and columns.	- Draw dots as the representations of geoboard's pegs. - Draw the dots either in rows or in columns. - Draw the dots only on the edge of rectangular array.

Result and Discussion

As mentioned in the MATH GASING Method, concrete problem was used as a starting point and introduced in a fun manner. Therefore, the eggs problem was chosen as the problem in which the students could gain more insight about the idea of multiplication. The explanations of each activity are as follows.

a. Counting the Number of Eggs

In the first activity, the students were asked to find the number of eggs in the picture (figure 3). It was expected that this activity could provoke students to do several counting strategies such as counting by ones, skip counting forwards, and repeated adding.

Figure 3. Eggs problem: "How many eggs altogether?"

Counting by ones is the early conceptual mathematics of counting. In this skill, the students should recognize that each counting number identifies a quantity that is one more than the previous number and that the new quantity is embedded in the previous quantity [8]. This strategy of counting was used by Depeton. He counted the number of eggs per row as shown in the figure below (figure 4).

Figure 4. Depeton counted the number of eggs by using counting by ones strategy.

As students investigated multiplicative context, their initial strategy of counting by ones become inefficient [2]. Because this strategy is so tedious, students construct better ways to keep track. They came up with the idea of skip counting. This idea appeared from a student named Linus. When he found the idea of skip counting by five, we tried to provoke him also other students to come up with the idea of repeated addition as recorded in the conversation below.

Linus : *Five, Ten, Fifteen, Twenty, Twenty five, Thirty.* (pointed out one egg in each column)

Researcher : *Wait, which one is 5? This one?* (pointed one egg in the

Linus and : leftmost column)
other *No, this one, altogether in this row. One row consists of 5*
students : *eggs.* (Linus pointed out 5 eggs in the first column)
Researcher : *Okay, can you recount it again? Show me which one is 5.*
Linus *Five, this all* (pointed out eggs in the first column) *is five,*
add by this (pointed out eggs in the second column) *and it*
makes 10, add by this (pointed out eggs in the third
column) *and it makes 15, add by this* (pointed out eggs in
: the fourth column) *and it makes 20, add by this* (pointed
Researcher : out eggs in the fifth column) *and it makes 25, and 30.*
All students : *How can you count it so fast? Five and then ten, fifteen…*
Siors *Add them!*
Five plus five plus five plus five plus five plus five.

From the conversation above, it was observed that the students could gain more insight about repeated addition which would lead them to the idea of multiplication. However, from this conversation, we could also know that the students were not aware the differences of column and row of an arrangement. Therefore, the next activity was designed to guide the students to be aware of equal objects in rows then connected it with array situation.

b. Counting the number of pegs which are formed from a rectangular array on the geoboard.

In this activity, the students were asked to make a representation of eggs problem by using geoboard and were guided to perceive the idea of multiplication from array structures (figure 5).

Figure 5. Students represented the structure of eggs appeared on the screen to Geoboard.

From the activity, the students could grasp the idea of repeated addition in the dimensions (rows or columns). They were able to use the pegs in Geoboard as indicators of how many rows or columns and how many pegs in each row or column. At this moment, students' spatial structuring of arrays come as a result of their organizing actions (motor and perceptual) [1] on the sets of pegs in each row or column. That is, students created spatial structures for sets of objects through mental actions they perform on the objects. Fosnot and Dolk [2] revealed that in this stage, students do not "read off" these structures from objects, but instead, employ "constructive structurization". This constructive structurization emerged because arrays were used. There was an implicit psychological suggestion presented that might facilitate the construction of skip counting strategy. Students idea of 5 rows consist of 6 pegs to represent eggs problem situation was used as the starting point to introduce the idea of multiplication, namely 5×6.

Next activity, after the students got the idea of multiplication from array structure, they were asked to construct rectangles which represented the form of multiplication of two numbers. They did not have any difficulties in making rectangles which represented multiplication numbers less or equal than 11. It was because the number of pegs in each rows and columns is 11. Cognitive conflict occurred when the students were asked to construct 20×3. Most of them were argued that they could not make 20×3 by using their Goeboard. After a while, a student, Martinus, came up with the idea of combining two Geoboards to represent 20×3 (Figure 6). It indicates that Martinus and other students who agreed with his idea could connect the idea of 20×3 as 20 rows consist of 3 pegs.

Figure 6. Martinus idea of combining two Geoboards to construct 20×3.

c. Drawing array structures which represent multiplication problems.

In this activity, the students were asked to draw dots, as the representation of pegs, in a rectangular array to represent some multiplication problems. They were given six multiplication problems, namely: 8×7, 9×9, 1×1, 5×1, 2×8, and 1×6. There were some strategies used by the students to draw the representation of array structures in the geoboard.

One student drew the 'perimeter' of a rectangular, then filled the inner part. Some students draw dots row by row while others do column by column (figure 7).

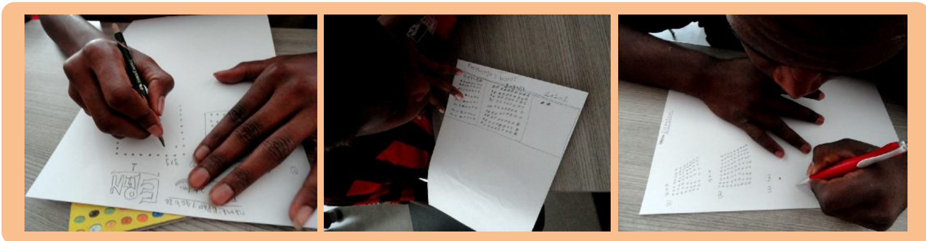


Figure 7. Students' strategies in drawing array structures which represent multiplication problems.

Most of the students could draw 8×7, 9×9, 2×8 without any difficulty. However, most of them were unsuccessful in drawing the representation of 1×1, 5×1, and 1×6. Although the idea of $a\times b$ which means *a* rows consist of *b* columns had already learnt by the students from the previous learning sequence, most of them drew either 1×5 instead of 5×1 as shown in the figure 8**.** Only two of twelve students were successfully drawing the representation of 5×1 as 5 rows consist of one dot in each row.

Figure 8. Students' drawing of 5×1.

From this finding, we would argue that the students had constructed that the arrays stayed the same, even though one's perspective for looking at it might change (the rows and columns) [2]. This idea would lead them to the idea of commutative property, namely $5\times1=1\times5$.

Conclusion

As other results of many studies, the conclusion cannot be drawn generally to contribute for future researches. In such cases, students' differences on thinking and the given treatments also give specific influence for the limitation of the results. The object of this study, then, provided some insights into how rectangular array model supported students' spatial structuring in learning multiplication.

Taking an overview of our findings from the classroom observations, what we found was that the rectangular array model for multiplication did indeed support calculation strategies. This ranged from simple counting strategies, counting by ones, through to identifying groups within the array and counting the numbers or rows or columns.

Because of its structure, the rectangular array model provides a clearer theoretical link to the important properties of multiplication in which could link to a 'real-life' situation for multiplication. Starting from concrete situation, students could see the structural similarities of arrays and created spatial structures for sets of objects. Therefore, we would suggest that the use of the rectangular array model be promoted more as key representation for multiplication, and students be given the opportunity to recognize the rectangular array as a representation of multiplication.

Having carried out this study, there are number areas that we would wish to investigate further. In this study we have suggested instructional activities incorporating the rectangular array model. We may wish to develop this instructional activity further with teachers and examine the impact of implementation of such a sequence on primary students. This may include looking more at students' ability in connecting the numerical values of some multiplication problems with its spatial representation and other models for multiplication as well as the rectangular array model.

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